A LABORATORY COURSE IN PHYSIOLOGICAL PSYCHOLOGY.

BY EDMUND C. SANFORD.

(*Fifth Paper.*)

THE VISUAL PERCEPTION OF SPACE.

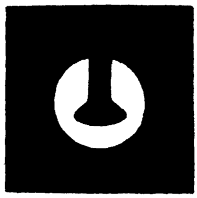
It is upon this field more than any other in physiological psychology, unless it be that of the psycho-physic law, that discussion has been most protracted and the accumulation of evidence most imposing. A complete treatment of the question involves arguments from surgery, pathology and other sources quite outside the possibilities of the laboratory. And even then, it is difficult, if not impossible, to establish one theory surely as against all others. Apart from the question of original sensations, however, there is a certain degree of harmony among investigators, and it is the commonly accepted experimental facts that this section of the Laboratory Course aims to gather up. The discussion of the ultimate matters may be followed in the works of Helmholtz, Hering, Stumpf, James, Wundt, and others. For the facts of spatial vision in general, see the works of Helmholtz, Hering, Aubert, Wundt, James and Le Conte. For special facts, special references will be given below. The subject is also more or less fully treated in the standard physiologies, Bernstein's Five Senses, McKendrick and Snodgrass's Physiology of the Senses, and other books of the same kind.

The ordinary seeing of space in its various aspects of distance, direction and size, rests on the retinal and kinæsthetic sensations of both eyes. And in every normal act of vision all of these sensations are either present themselves or by their reproduced images, and this fact must not be forgotten. For the sake of simplicity, however, it is necessary to separate the factors more or less completely, and to treat now of one and now of another. Those phenomena of which the presence of two eyes is an essential condition form a class by themselves, and will be reserved for a special section; the present one will be given to a portion of the facts of monocular vision, or rather to some cases in which the presence of a second eye is unimportant.

MONOCULAR PERCEPTION OF SPACE.

156. The Outward Reference of Visual Perceptions. The outward reference of visual perceptions probably comes about through their co-ordination with those of other senses, especially those of tactual and kinæsthetic origin, but the matter is too complex for direct experiment. It is easy, however, to study the relations of the retinal image and the outer objects that produce it, and much has been written on the outward projection of retinal states. It

must, however, be kept clearly in mind that retinal states as such, are never perceived, and especially that retinal sensations are not first given a location in the eye, and then at some later stage transferred outward. Considered physically, the retinal image is reversed with reference to the objects that it represents. This has already been seen for the rabbit's eye in Ex. 99, and for the human eye with Purkinje's vessel figures and the phosphenes (Exs. 107 and 113). It can be shown also in the following experiment with retinal shadows:



Le Cat's experiment. Hold a pin, head upward, as close as possible before the pupil, and an inch or two in front of the pin, a card pierced with a pin-hole. Move the pin about till it comes into exact line with the hole, when there will be seen in the circle of diffusion representing the hole a shadowy inverted image of the pin-head, somewhat as appears in the accompanying cut. The rays of light from the pin-hole are too divergent to be brought to a focus on the retina, but enter the eye in a favorable state for casting a shadow. The shadow on the retina is erect like the pin that casts it, but is perceived inverted in its outward location. Observe at the same time the still more blurred, erect image of the pin through which the other things are seen. This is not a shadow but a true retinal image formed in the ordinary way by light reflected from the surface of the pin. When several pin-holes are used (three at the points of an eighth of an inch triangle, for example) an equal number of shadows will be seen.

The casting of the shadow can easily be illustrated with a candle, a double convex lens and a bit of card. Set the lens a foot or two from the candle and hold the card too near to the lens for the formation of an image, then introduce a finger or pencil close before the lens on the side toward the light and observe the erect shadow on the card.

Le Conte, *B;* Wallenberg, Laqueur.[1]

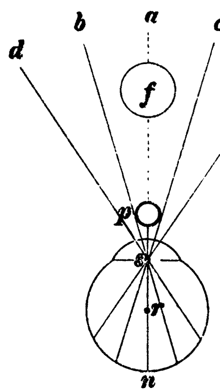
157. Monocular perception of direction; directions from the eye. The perception of direction is ordinarily binocular, and the centre to which directions are related lies between the eyes, even when one is closed. This will be proved in the experiments on binocular perception of space. The binocular perception, however, must rest on a perception of the relative direction of points in the monocular field, and this will be considered in the next few experiments.

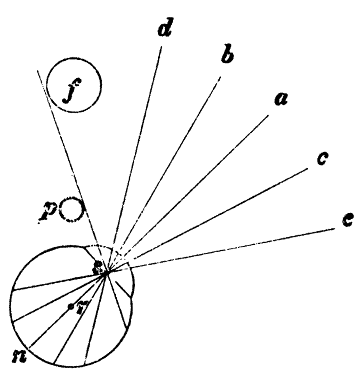
Two luminous points appear to have the same direction when one is exactly covered by the other, or, to state the matter in retinal terms, when the image of the one for which the eye is accommodated lies in the centre of the circle of diffusion of the one for which the eye is not accommodated, or, if both appear in diffusion circles, when the centres of these circles coincide. The lines

[1]See bibliography at the close of the article.

drawn through points in this relation and prolonged to the retina are known as *sighting lines* (*Visirlinien*, *Lignes de visée*), and cross in the centre of the pupil, or rather, in the centre of the image of the pupil formed by the cornea, about 0.6 mm. forward of the true position of the pupil and 3 mm. from the summit of the cornea. These lines might well be called "lines of direction," had not this name been already given to another set of lines, namely those which are drawn from the points of external objects to the corresponding points of their retinal images. These have already been mentioned in Exs. 101 and 112, and they give with certain limitations the directions in which objects appear when the eye is exactly accommodated for them. Their point of intersection is about 7 mm. from the summit of the cornea. They are important for optical purposes, but for the general perception of direction are less important than the sighting lines, though for remote points and for points near the fixation point, the differences between the two sets of lines is very slight. For points remote from the fixation point, for reasons to be given in a later experiment, neither set of lines gives the direction in which objects are seen.

The position of the crossing point of sighting lines is found by inference from the optical structure of the eye. To make a sure empirical determination would be laborious, but it is easy, however, to show that the crossing point is considerably in front of the centre of rotation of the eye (about 10.6 mm.). Place a candle at a distance of a foot or foot and a-half from the eye. Look toward the flame with a single eye, but hold close before the eye a pencil or narrow strip of black cardboard. So long as the eye looks straight forward, the flame is entirely hidden by the pencil. When, however, the eye is turned strongly to either side, the flame instantly appears on the side toward which the eye has been turned. The explanation of this will readily be seen from the following diagrams in which p represents the pencil, f the flame,





s the centre of sighting lines, and r the centre of rotation. The lines radiating from s are sighting lines, sa being the principle one, which is practically coincident with the line of sight.

Helmholtz, *A*, F, 692 (539), 745 ff. (584 ff.); Aubert, *A*, 461 (on the sighting lines).

158. Monocular perception of direction; directions in the field of vision. The relative direction of points in the field of vision

cannot be changed, of course, without changing the direction of the points from the eye; it is easier, however, to experiment on points in the field.

a. Lines that appear straight in indirect vision. Lay a large sheet of paper on the table and mark a fixation point in the middle of it. Two or three inches to one side of the fixation point place a button or bit of black paper, and, a foot above and below, other buttons or bits of paper. Then leaning over the table so as to bring the eye above the fixation point try to place the three buttons in a straight line, holding the eye steadily upon the fixation mark. Examination of the buttons when placed will show that the middle one is too near the fixation mark, *i. e.*, the attempt to make a straight line has resulted in a curve convex toward the fixation mark. Try also with the buttons in a horizontal line.

b. Actual straight lines in the periphery of the field. If lines convex toward the fixation point appear straight, lines that are actually straight should appear concave. On a large sheet of paper draw a pair of parallel lines three or four inches apart and two or three feet long. Place a fixation point half way of their length and half way between them; fasten the paper to the wall or spread it on the table, and observe as in *a* above. Try with the lines both vertical and horizontal and in oblique positions. The same thing in part, at least, may be noticed in the double images of a single vertical thread seen binocularly.

In a spherical field of vision, the parallel lines of experiment *b* would be represented by great circles, the horizontal pair for example, having their poles at the right and left ends of the horizontal axis of the field, their planes making equal angles above and below the plane of the horizon.[1]

It is obvious that changes in direction which make straight lines appear curved cannot take place without introducing slight errors of distance also. The shortest distances for perception are the curves which appear straight.

c. Nature of lines that appear straight in indirect vision. It would, of course, be possible by developing the method used in *a* above to make a somewhat exact study of the nature of the lines that appear straight in indirect vision, but their general nature may be found in another way. In the hemispherical field of regard these lines are circles — Helmholtz's *Circles of Direction.* The following diagram shows the projection on the visual field of a system of these circles of direction. For use the diagram must be enlarged five or six times. It should be viewed with the single eye opposite its centre, and at a distance proportionate to the length of the short line below the diagram. In order to fix this distance, it is convenient to cut a small rod of such length, that when the eye is at the right distance the rod will just extend from the outer edge of the socket of the eye to the diagram. When the head is brought into the proper position and the eye is fixed on the middle of the diagram, the lines of the figure will appear approximately straight and parallel. Try with the diagram in the position shown below, and also when turned so as to make the principle lines oblique. Especial care should be taken to avoid movements of the eyes, for a new interpretation of the curves is thus introduced, and the checker-board seems concave instead of plane. Some dis-

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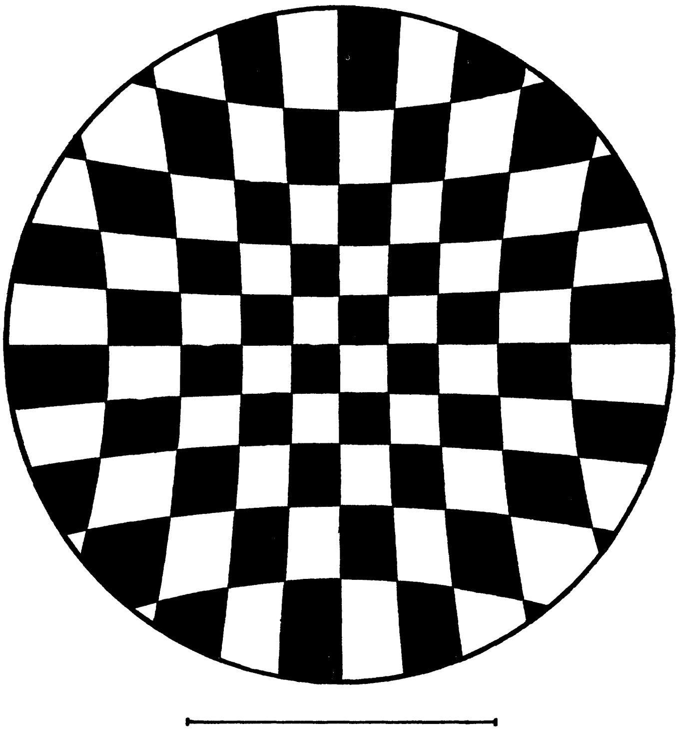
[1]It should not be supposed from this that the naïve field of vision is hemispherical. It is neither definitely hemispherical nor definitely anything else, except as it is formed by the conditions and habits of vision. It is spoken of as a plane or as a hemisphere for greater ease in exposition.



advantages are escaped by fixating the centre of the diagram till a sharp and strong after-image is secured, and then observing this with closed eyes turned toward the sky.

After getting the general effect, the observer should repeat the observation beginning first at a distance (greater than that just used) at which the direction of curvature in the lines can easily be recognized, and then slowly decreasing the distance till a point is reached where the lines seem straight and the squares equal, and still further till the curves appear to bend the other way. Test the distance at which the lines seem straight with the little rod mentioned above ; it will generally be found to agree approximately with the distance for which the diagram is calculated.[1] The projections of the circles of direction are then the lines that seem straight in indirect vision. These circles of direction are lines along which the eye (when moving according to Listing's Law) can carry a short after-image without causing the image to leave the line, in other words they are, for the eye in motion, straight lines ; and the experiment shows that even when the eye is kept still, its

[1]The agreement is not absolutely perfect, and there are perhaps, in addition, individual differences depending on the exactness with which the eyes follow Listing's Law. Helmholtz finds the curvature of the extreme verticals on the temporal side a little too great, and Küster working by a slightly different method appears to have found it too great for all the curves. (See Hering, *A*, 370, foot note.)

experiences of movement exercise a controlling influence on its perceptions. (For a fuller account of Listing's Law and the circles of direction, see Ex. 123, and the Appendix at the end of this section.)

d. Illusions of form in indirect vision. Radial distances, as the diagram of *c* shows, are more decidedly under-estimated than distances parallel to the margin of the field. This is easily shown by laying a large sheet of paper on the floor—a strip three feet long and a foot wide answers well, when the narrow side is next the observer—and standing so that when the eyes are directed horizontally forward, the paper will be seen at the lower edge of the field. When the eyes are turned from the horizon to the paper, the latter noticeably increases in width (*i. e.*, in a direction to and from the observer) when the eyes are again directed to the horizon, it suffers a corresponding contraction. Changes in the other direction are hardly noticeable. Disks of paper (three to six inches in diameter) when viewed in indirect vision appear as ellipses with their short axes directed toward the fixation point. For the part of the field near the fixation point such illusions as this and those of *a*, *b* and *c* are so slight that they may be neglected.

The whole field of vision itself appears narrower than it really is; it actually covers an extent of nearly 180°, and yet under favorable circumstances, as when looking at the dark field of the closed eyes, or at the sky in the absence of all landmarks, the extent may seem not much over 90°.

Helmholtz, *A*, F, 706-718, (551-561). Wundt, *A*, 3te Aufl., II, 112-114; 4te Aufl., II, 128-130. Hering, *A*, 369 ff., 535 ff.

159. Monocular perception of direction; directions in the field of regard. The observation that the perceptions of the eye at rest are modified by those of the eye in motion, is still further confirmed by the similarity of other phenomena of the field of regard and the field of vision.

a. Straight lines viewed with the eyes in secondary positions. Experiment with a single eye and a long ruler held horizontally before an even wall space or other uniform background. Hold the flat side of the ruler toward the face and about a foot distant from it. Try first with the ruler eight or ten inches above the primary position of the line of sight (cf. Ex. 123), running the eye freely back and forth along the edge, and observe that the edge appears curved upward, *i. e.*, concave below. Try with the ruler depressed a somewhat greater distance below the primary position and observe the contrary curvature. Try also with the ruler vertical and to the right and left. Little advantage will result from too extreme positions of the ruler. The curvature to be observed is not very great, but that it is due to the visual apparatus and not to the ruler, is easy to show by turning the rule over, which would reverse the direction of curvature in the ruler, but not that of the curvature which depends on the eye. Change of position of the ruler from above to below the primary position of the eye, on the contrary, reverses the direction of the curvature due to the eye, but not a real curvature of the ruler. Compare the results here found with those in Ex. 158, *a* and *b*.

The occasion of the illusion is the rotation of the eyes when moved from point to point in secondary positions. (Cf. Ex. 123, and the Appendix at the end of this section.) When the eye is kept fixed on the end of the ruler, or moved slowly, the ruler may seem slightly tilted instead of curved.

b. The apparent vertical. For the single eye a true vertical appears to incline a very little inward, *i. e.*, to the left for the right

eye and to the right for the left eye. Place on the field of the campimeter a large sheet of paper, and on it draw an exactly horizontal line at about the height of the eye when the observer is in position. Exactly above the point of this line to which the line of sight is naturally directed, set a tack and hang from it a black thread 60 cm. long with a weight at its lower end. At a distance of 57.3 cm. from the tack, draw a second horizontal line and paste along below it a bit of millimeter paper. Let the observer take his position and carefully push the weight to one side or the other (as may easily be done with a needle mounted in a short handle of wood) until the thread seems to be exactly vertical, and the angles that it makes with the horizontal line exactly right angles; then, let him hold the thread in place by sticking the needle into the wood and note the amount of the angle from the millimeter scale—on which, if the dimensions above have been observed, 5 mm. will correspond to 1°. In this experiment the eye may be moved up and down the thread as desired.

Repeat the experiment, keeping the eye fixed in the primary position. The amount of the inclination necessary has been found by Donders to be decidedly variable even in the same observer. The attachment of the eye muscles is such that with elevation of the lines of sight there is a slight turning outward, and with depression a slight turning inward. A line which the eye follows exactly in this upward and downward movement, *i. e.*, a line inclined a little outward seems vertical.

On *a* see Helmholtz, *A*, F, 699 (545); Hering, *A*, 536. On *b* see Helmholtz, *A*, F, 700 ff. (546 ff.); 716 f. (559 f.); Wundt, *A*, 3te Aufl., II, 122 ff.; 4te Aufl., II, 140 ff.

160. The tendency of the eye to follow lines and especially contours. This tendency is of importance in the seeing of form, because it results in clear vision of the object and because it complicates the whole matter by introducing kinæsthetic factors. It is a habit, however, that is not beyond conscious control, and for that reason is more difficult to demonstrate by overt experimentation than by casual observation. Any one that will take note of his own seeing when presented with objects with strongly marked lines, will easily find trace of the habit. In imagining geometrical figures, also (for example, an eighteen-inch hexagon drawn on the blackboard) something of the same tendency will often be found. The following experiment aims to give a laboratory means for making such observations.

Paste upon a piece of cardboard eight and one-eighth inches long and four inches wide, two four-inch squares of red paper in such a way as to cover all the card, except a white stripe one-eighth of an inch wide between them; cover the whole with a sheet of semi-transparent paper as for Meyer's experiment (Ex. 142 *c*). Examine the white stripe for the effects of contrast. After the examination has lasted a few seconds, suddenly lay across the middle of the diagram a bit of wire six or eight inches long, approximately at right angles to the white stripe. If the experiment succeeds, the white stripe will instantly show a strong increase in the complementary color. Before the introduction of the wire, the eye is chiefly engaged in following up and down the white stripe, and the contrast effects are confined to those of simultaneous contrast. When the wire appears, the eye changes to it and moves back and forth along it once or twice, and thus brings upon the white stripe the more powerful effects of successive contrast.[1]

[1]This experiment originates with Waller (Journal of Physiology, XII, 4, p. xliv), but is used by him for a totally different purpose.

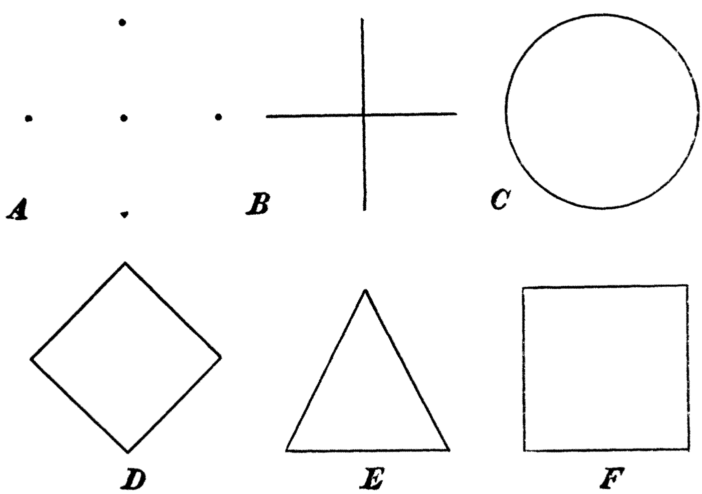
GEOMETRICAL ILLUSIONS.

Besides the illusions just considered, there are a large number of others that show in greater or less degree the influence of eye motions in the formation of visual perceptions. It is very likely that many of them — even those that appear simple — are the resultant of many other experiences than those of ocular motion, but in most of those that are to be given here, eye motions are certainly an important factor. They fall into classes according to certain empirical principles and have been so grouped below, but merely for convenience and without any intention of prejudicing their explanation. Typical examples only are given, for the number of variants of some of them is very large.

In all of them the student will do well to turn the diagrams about and to view them from different sides so as to separate the illusions that depend on position from those that are independent of it. In general, illusions are strengthened when the affected lines are made oblique in the field, corresponding with the less frequency and certainty of eye movements in such directions. For the most careful study of these illusions they should be separated from one another, and from the influence of all extraneous lines, *e. g.*, drawn singly on good sized sheets of paper.

161. Illusions in the perception of distances depending on their direction in the field of vision.

Vertical distances are over-estimated in comparison with horizontal distances. Lay off by eye on a sheet of paper placed vertically before the face equal distances, up, down, right and left, from a central dot, marking the distances by dots as in *A* in the accompanying diagram. Repeat several times and measure the distances found. In the diagram all the vertical and horizontal distances are equal, and in all cases, except in the circle, the vertical seems too long.



In *D* the over-estimation of the height entails an under-estimation of the angles above and below, and an over-estimation of those at the right and the left. The illusion is less in the line

figures and absent entirely in the circle, in Wundt's opinion (4te Aufl. II, 152), because in the case of these familiar figures, perception is influenced by knowledge of the geometrical relations of the parts. (For quantitative studies see among others, Kundt, and Münsterberg, 164 f., 175 ff.) A similar, though slight, difference is often found between horizontal distances to the right and left, when careful experiments are made with the single eye. (For quantitative measurements under various conditions, see Kundt, and Münsterberg, 160 ff.)

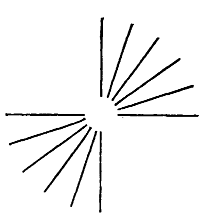
Distances in the upper part of the field are over-estimated as compared with those below them. This illusion may be tested actively as follows: Near the top of a strip of paper twelve or fifteen inches long and five or six wide, draw a horizontal line two inches long. Take this again as a standard and draw half an inch below it a second line of a length that seems equal to the first. Then cover the first line and taking the second as a standard, draw a third and so on, continuing this process till the strip is full. Then uncover and measure the first line and the last. With this Wundt associates the S's and 8's which seem a little smaller at the top than at the bottom when in their usual position, but a good deal larger above when inverted: S 8.

For all of these illusions Wundt finds an explanation in the differences of effort required for turning the eye in different directions (3te Aufl., II, 119 ff.; 4te Aufl., II, 137 ff.). The superior and inferior recti are relatively weaker than the external and internal muscles of the eye. Furthermore in elevating or depressing the eyes, the oblique muscles partly oppose the superior and inferior straight muscles, and so render a portion of the effort ineffective for purposes of motion. Why lines in the upper part of the field should seem longer than those lower down, is not specifically stated, but by implication it also is to be credited to muscular differences.

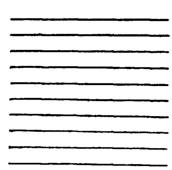
For references see those given at the end of Ex. 167.

162. Illusions of interrupted extent. Interrupted intervals

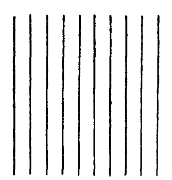
A



B



C



D

E

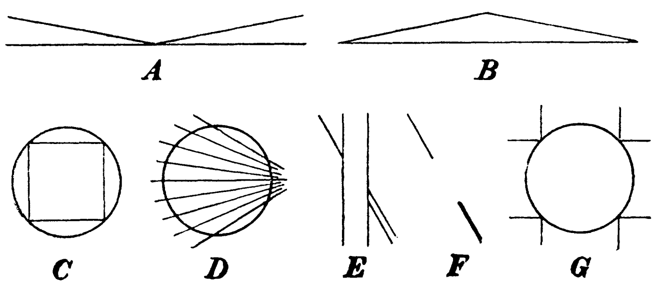
generally seem larger than free intervals. In the first two figures above, the interrupted extents seem larger than the free extents. In the last case, however, where the interrupted space has but a single dot in the middle, the principle suffers an exception. The

figure showing the filled angles should be regarded binocularly; monocular observation complicates the effect by introducing the illusion of Ex. 159, *b*. *C* and *D* are equal squares.

Here as before Wundt's explanation rests on variation in eye movements (3te Aufl., II, 126 f.; 4te Aufl., II, 144 f.). In passing over interrupted extents, the movement of the eye is made more difficult by the short stages into which its course tends to be broken, while it sweeps with relative freedom over the empty spaces. The fact that a single interruption in the middle of a space has an opposite effect, is explained by a tendency of the eye, when the middle of an extent is marked, to take in the extent simultaneously by fixation of the middle without motion. For other explanations see Helmholtz, *A*, F, 720 f. (563 f.).

For references see those given at the end of Ex. 167.

163. Illusions affecting the apparent size of angles. Small angles are relatively over-estimated. The figure with filled angles in Ex. 162 above might be classed under this principle as well as under that of interrupted extent, but in other figures the effect of small angles is seen by itself.



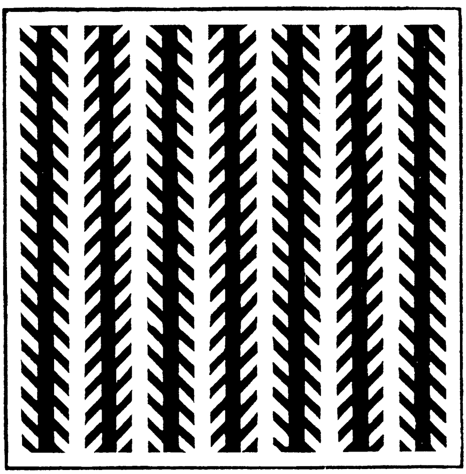
In *A* and *B* slight distortions are found in the horizontal lines. In *C* the circle is flattened at the corners of the square, and the sides of the latter are bent inward. In *D*, also, the distorting effects of the cross lines are unmistakable. In *E* the oblique line on the left is the real continuation of the lower line at the right, not of the upper as appears to be the case. This illusion is strengthened by viewing it from a distance, *i. e.*, by reducing the size of its retinal image. In *F* it is shown that the presence of an actual oblong is not essential to the illusion, though here, as elsewhere, the place of actual lines may be supplied more or less consciously by imaginary ones, or by the lines of other figures, the edges of the page or any other prominent lines in the field. In *G*, however, the explanation by the over-estimation of small angles completely fails, for the effect is the same in kind as in *C*, when the reverse was to be expected.

The over-estimation of acute angles Wundt refers also to eye movements (4te Aufl., II, 146). In figure *A* above, for example, as the eye follows the horizontal line toward its intersection with the oblique lines, it suffers an increasing attraction, as it were, toward the oblique line nearest it, and from this results the wrong conception of its route. The fifth figure according to Wundt involves several illusions. The figure is estimated too large in the direction of its prominent (vertical) lines, and in this the habitual over-estimation of verticals helps. When the latter is

excluded by turning the figure on its side, the illusion that remains is to be explained partly by the over-estimation in the direction of the prominent lines and partly by over-estimation of the small angles. In explanation of the first three figures, Helmholtz again cites his principle "according to which acute angles, being small magnitudes clearly limited, seem in general relatively too large, when we compare them with undivided obtuse or right angles," but believes that this principle yields in importance to ocular movements, if indeed, it does not itself depend on such movements (*F*, 724, 725, 726, 727, (566, 567, 568)). Cf. illusions depending on motions of the eyes, Ex. 168. In the fifth figure he thinks movement is less important, and that irradiation may co-operate (*F*, 723, (565)). The over-estimation of small angles is rejected as a principle of explanation by Lipps.

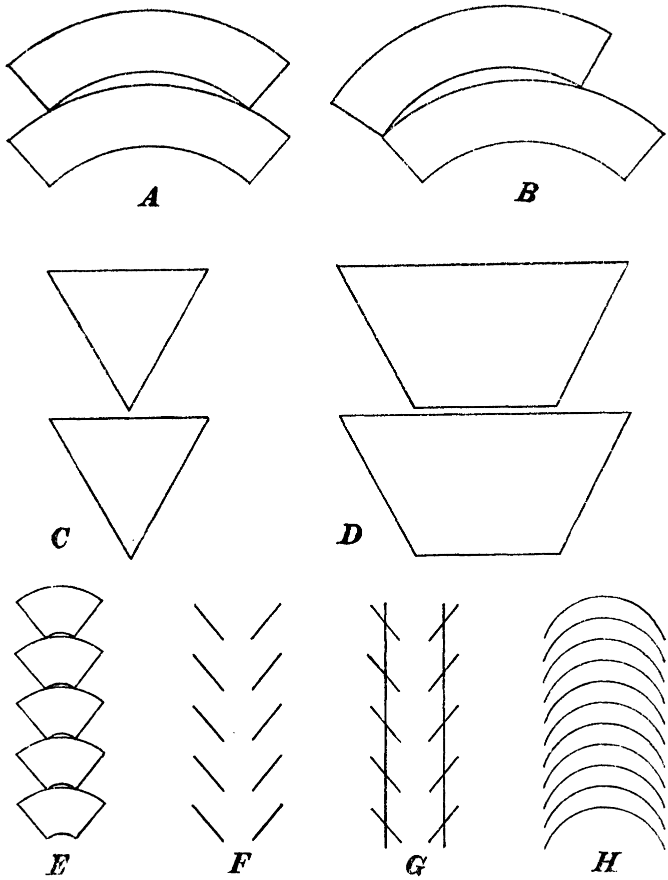
For references see those given at the end of Ex. 167.

164. Zöllner's figure. In principle this much discussed figure is the same as those of Ex. 163.



The oblique cross lines, making small angles with the verticals, cause an apparent dislocation of them in the same direction that they would be dislocated if the acute angle were enlarged. The greater effect in this figure would then be due to the multiplication of the small angles. For a partially concurrent explanation see Ex. 168; for divergent explanations see among others, Hering, Kundt, Lipps. The strength of the illusion depends on the angle of the oblique lines. For himself Zöllner found it greatest at about 30°. The short cross lines themselves show the illusion of *E* in Ex. 163, the right and left portions seeming not to be continuous.

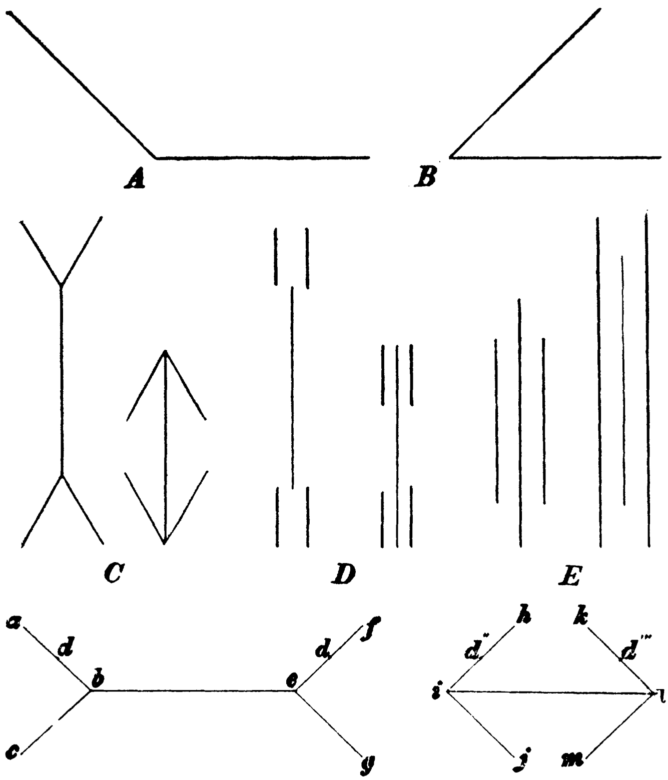
An interesting case in which Zöllner's figure is present, though masked, is that of the superposed segments of a ring. In *A* and *B* below, the upper one, in each case, seems smaller, though all are of precisely the same size. Müller-Lyer recognizes this as a case of the Zöllner's figure and suggests a transition to it by such figures as *E*, *F* and *G*. Wundt denies this explanation (4te Aufl., II, 151-152), asserting that if this were the case, the effect ought to



be the reverse, *i. e.*, the upper segment should seem larger, and gives *H* in which the upper curves seem a little larger in proof, but he evidently has only considered the curves and not the straight line at the ends of the segments. His own explanation which traces the illusion to association with cases in which the segments are referred to the same centre is clearly imperfect, for it does not fit the cases of the triangles and trapezoids, *C* and *D* above, where the illusion though weakened is still present. The fact may be that the ring segments involve co-operating illusions depending on both principles, while the straight line figures involve only one. The illusion is even more striking when the segments are cut out of cardboard, and can be shifted about and actually superposed.

For references see those given at the end of Ex. 167.

165. Illusions affecting the length of lines in the presence of other lines and angles. In *A* and *B* the sides of the larger angle seem

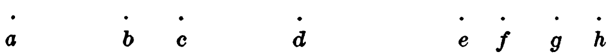


longer than those of the smaller angle. Try with the diagram in different positions to avoid the tendency to over-estimate vertical distances noticed in Ex. 161 above. In *C* the central vertical lines are of equal length, but that in the longer figure appears distinctly longer. In *D* and *E* the same is true, though the illusion is somewhat weakened.

A number of explanations for these illusions have been advanced, no one of which appears so exclusively right as to exclude the rest. (1) It has been held that figure *C* is only a development of *A* and *B* and that the angles modify the apparent length of their enclosing lines (Müller-Lyer, Jastrow). (2) It has been suggested that we unconsciously take into account the spaces adjacent to the lines to be compared; *e. g.*, the trapezoidal spaces *a b e f* and ***h i k l*** inclosed by the outwardly and inwardly inclined arms in the typical figure (Müller-Lyer and Auerbach). (3) Another writer considers that the tendency to over-estimate small angles and under-estimate large ones causes an illusory rotation of the short arms about their middle points, and thus a lengthening of the central line in one case and a shortening in the other. If, for example, *a b* and *e f*, were rotated in the sense required by this illusion about *d* and d', their ends *b* and *e* would be brought farther apart. Similarly, but with opposite effect, if *gh* and *kl* were rotated about

their middle points (Brentano). (4) Others have thought that the eye in moving along the central lines tends to follow out upon the short lines when they are outwardly directed, and to stop short of the end of the central line when they are inwardly directed (Delbœuf, Wundt, 4te Aufl., II, 149 f.), and in a way Lipps.[1] This explanation is supported as against (3) by the persistence of the illusion in such figures as *D* and *E*, where there are no actual small angles. In *E*, however, they may well be imagined. (5) It has been held again that these figures are in reality like *D*, *E*, *F* and *G*, Ex. 167, and that we do not really estimate the distance from the apex of the short lines, but from the centre of one group of lines to the centre of the other, from the centre of group *a b b c* to the centre of the group *fe eg*, each perhaps being regarded as a triangle with an imaginary side (Brunot). The skillful observer will very likely find several of these tendencies appearing in his own study of the figure, and the explanations are not totally exclusive of one another. If the over-estimation of small angles depends as Helmholtz thinks is possible (F, 730, (571)) on eye-movements, the third and fourth explanations might find a common ground. Perhaps, also, the tendency to estimate the distances from the middle of the short line groups, has its effect by influencing the movement, as might also the tendency to take into account the adjacent areas, though less obviously. In any case, the motor factor is an important one.

166. Illusions showing the effect of adjacent extents.[2] In cases *A*, *B*, *C*, *D* and *E*, the extent of the middle quadrilateral, angle or line seems smaller when it lies between large extents than when it lies between small extents. Müller-Lyer seems to hold that the observer first compares the extents in question with their surroundings and then with each other.[3]



This process might be expressed schematically somewhat as follows:

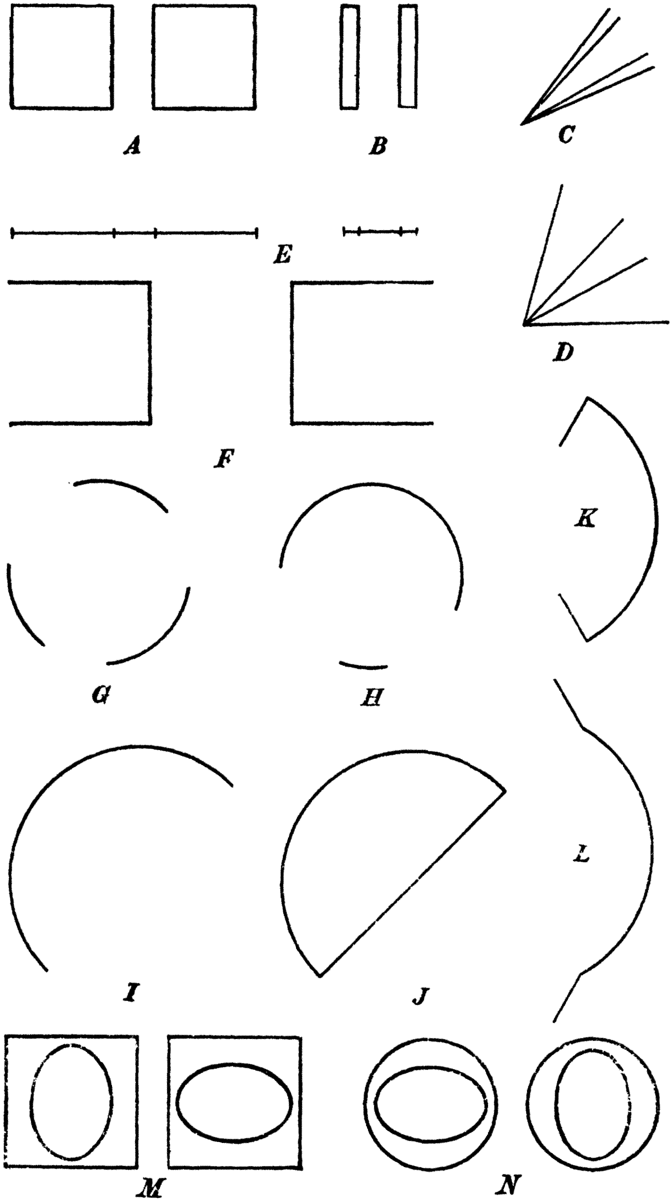
bc	in comparison with	*ab*	or *cd*	is small.
fg	" "	*ef*	or *gh*	is large.
bc	" "	*fg*		is small.

In a similar way the sides and bases of parallelograms are said to be mutually affected, and even parallel lines seem nearer together or further apart as they are long or short. In this case, which is the only one noticed by him, Wundt (4te Aufl., II, 146–147) explains the illusion by the strong tendency to move the eyes lengthwise along the long parallels, which results in an under-estimation of the figure in a vertical direction. In figure *M*, the squares are distorted horizontally and vertically by the inscribed ellipses. In *N*, however, the circles are distorted, if at all, in an opposite

[1]Lipps in his explanations makes use of many picturesque expressions with reference to these figures and their parts such, as, liveliness, inner activity, upward-striving (*Lebendigkeit, innere Regsamkeit, Emporstrebens*), thus seeming to attribute different effects to "forces" inherent in the figures, and for this he is criticised by Wundt. It seems probable to the writer, however, that if he had undertaken an analysis of the "forces" that he finds, he could hardly have avoided agreement with the eye-movement party. Indeed in his *Grundthatsachen des Seelenlebens*, p. 527, he makes use of eye-movements to explain illusions of the Zöllner type, though he regards them in the last analysis as psychic and not sensory illusions.

[2]To these illusions, Müller-Lyer gives the name of Illusion of Confluxion (*Confluxionstäuschungen*).

[3]Müller-Lyer, 266-267.



direction. In *M* we have a case of the influence of adjacent lines; in *N*, however, this effect is overcome by the greater power of the

illusion of Ex. 163, which here affects the diverging lines of the circle and ellipse. It seems possible also that we may have here at the points where the circles and ellipses are nearest together a compressing effect like that in the concentric circles in Ex. 167.

When the circumference of a circle is interrupted, the remaining arcs seem too flat to belong to a circle of such radius; so also, a semi-circle seems like an arc of a greater circle of less than 180° extent. Closing it by drawing the diameter makes it seem smaller. Cf. Figs. *G*, *H*, *I* and *J*.

This illusion Müller-Lyer connects with that noticed in Ex. 165, making a transition through figures *K* and *L*, the middle line being curved instead of straight. The short straight lines tend to lengthen and flatten one arc and shorten and bend the other. If short arcs of the same radius as the central ones be substituted for the short straight lines, and then be rotated till they form a continuation of the middle arcs, they might still be expected to produce their usual effect, from which would result the generalization that every portion of a circular arc influences the form of every other portion, in the direction of contraction in the complete circle and large arcs, and of expansion in the case of arcs less than 180°. Wundt, also, refers the case of the semi-circles to the same general principle as that of Ex. 165 (though his principle is somewhat different from Müller-Lyer's), but believes that the small arcs in *G* and *H* seem flat, because the movement of the eye in following them is not very remote from that for following a straight line (4te Aufl., II, 149 f., 151 f.).

When one side of a quadrilateral figure is removed, the figure seems too long in the direction toward the side removed and too short in the other direction. In figure *F* the three-sided squares seem too long in a horizontal direction, too short vertically. The reverse is the case with the space between them which is also a square of equal size.

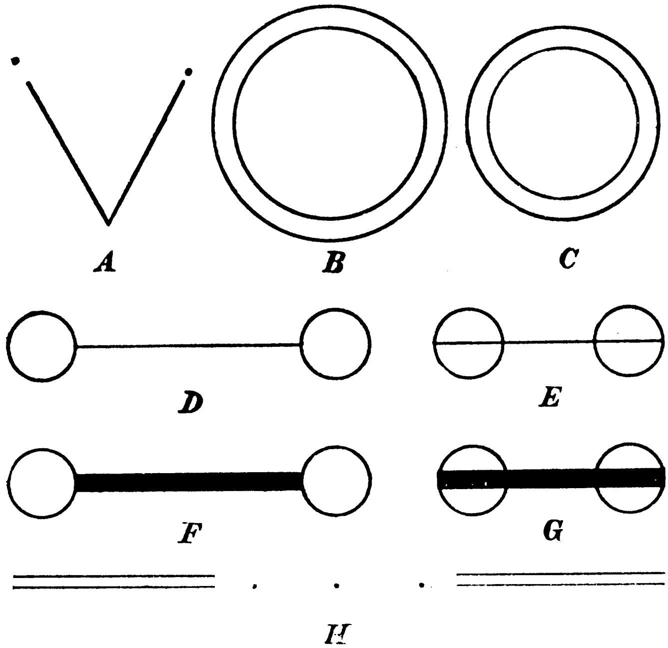
The explanation given by Müller-Lyer for this illusion is double. The open-sided square seems too long in the direction of the open side, because a certain portion of the free space is included in the estimate, and then it seems too narrow because it seems too tall.

167. Some unclassified geometrical illusions. An interesting figure from Láska is given as *A* below. The sides of the angle are equal, but are made to seem unequal by the setting of dots at unequal distances from them.

In *B* and *C* is shown an illusion from Delbœuf. The inner circle in *B* and the outer one in *C* are the same size, but that in *B* looks larger and that in *C* smaller.

In Delbœuf's opinion, the illusion depends on the interference of the extra circles with the measurement of the diameter by the eye. In *C* the inner circle holds the eye back, as it were, and in *B* the outer circle draws it on. Wundt's explanation (4te Aufl., II, 146-147), is here, as in the case of the parallel lines already mentioned, the under-estimation of distances in directions opposed to the chief tendency to movement. In this case the eyes tend to follow the parallel circumferences which causes under-estimation of the distance between them, making the larger seem too small and the smaller too large. If this tendency to movement is opposed by a fixation mark in the middle, he finds that the illusion disappears, as in the case of the space broken by a single central dot in Ex. 162.

In *D* and *E* is shown another striking illusion, again from Delbœuf. The distance between the adjacent edges of the left hand pair of circles is the same as that between the remote edges of the right hand pair, though the latter looks considerably less. The tendency



apparently is to estimate the distance not between the points mentioned, but between the centres of the circles in each case. In *F* and *G* where the heavy lines interfere with this tendency, the illusion is a little weakened. For these figures, Delbœuf's explanation is like that for *B* and *C* above; in *E* and *G* the eye is restrained, in *D* and *F* it is drawn onward.

In *H* is shown an illusion from Mellinghoff. The three dots in the free space between the parallel pairs seem a little above the level of the lower lines of the parallel, though they are not actually so. The upper lines of the parallel pairs, according to Wundt (4te Aufl., II, 146), attract the eye as it is swept across the figure, toward a position intermediate between the upper and the lower lines, and to this position the dots are assigned.

In the parallel columns below is shown another illusion of common experience with printers :

In these two columns the type is of exactly the same size. On this side, however, it is set "solid" and looks smaller than on the other. According to Wundt (4te Aufl., II, 150), this is because the eye passes over the same number of letters in a shorter course.

Here the lines are "leaded," *i. e.*, have greater space between them. Is it not possible that the illusion is based on the greater general whiteness in this case and blackness in the other ?

For a collection and discussion of a number of other illusions, some similar to those of preceding experiments and some different, see Lipps, *A* and *B*.

On the Geometrical Illusions in general see Wundt, *A*, 3te Aufl., II, 115-132, and 4te Aufl., II, 137-156. Helmholtz, *A*, F, 720-733 (563-573).

On Ex 161: Kundt, Münsterberg, 164 f., 175 ff. Wundt, *A*, 3te Aufl., II, 119 ff.; 4te Aufl., II, 137 ff. Helmholtz, *A*, F, 697 (544).

On Ex. 162: Wundt, *A*. 3te Aufl., II, 124 f.; 4te Aufl., II, 142 f. Helmholtz, *A*, F, 720 f. (563 f.). Knox. Watanabe.

On Ex. 163: Wundt, 3te Aufl., II, 125 ff.; 4te Aufl., II, 145 ff. Helmholtz, *A*, F, 722 ff. (564 ff.). Lipps. *B*, 269, 291. Jastrow, *B*. Dresslar.

On Ex. 164: Helmholtz, *A*, F, 722-725 (564-568). Wundt, 3te Aufl., II, 125 ff.; 4te Aufl., II, 144 ff. Zöllner, *A* and *B*. Lipps, *B*, 267 ff. Jastrow, *A*, and the literature cited by him.

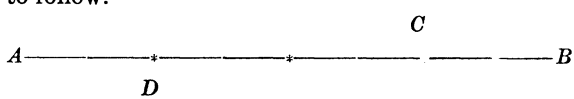
On Ex. 165: Müller-Lyer. Jastrow, *A*. Brentano. Lipps, *A*. Delbœuf, *C*. Wundt, *A*, 4te Aufl., II, 149 f. Brunot.

On Ex. 166: Müller-Lyer. Wundt, *A*, 4te Aufl., II, 149 f., 151 f.

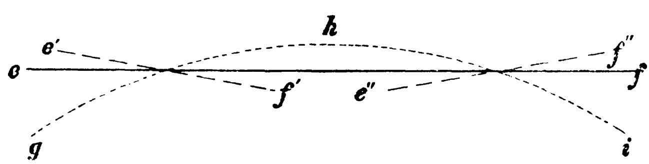
On Ex. 167: Láska. Delbœuf, *A* and *B*. Wundt, *A*, 4te Aufl., II, 146, 147, 150. Brunot.

168. Illusions depending directly on movements of the eye.

a. Move a pin head along the imaginary line *C D* in the figure below, keeping the eye constantly fixed on the pin head as it moves. The line *A B* will seem to move downward and to the left as the pin head goes from *D* to *C*, and upward and to the right as it goes from *C* to *D*. Steady fixation of the pin head is essential; a moderate rate of movement, which can be found by a few trials, gives the best result. The right and left movement of *A B* may be increased by moving the pin head in a line more nearly horizontal than *C D*. In this case movement of the image of *A B* on the retina is interpreted as movement of the line instead of the eye. For the fuller consideration of such cases see experiments on the perception of motion to follow.



b. An illusion affecting the direction of a line is to be observed when a compass point is made to draw an imaginary arc, cutting the line *e f* as the dotted arc *g h i* does in the figure below. As the point advances from *g* to *h* the line appears to take a position like that of *e' f'*; as the point traverses the region about *h* there is a sudden change, the line inclining now in the direction of *e'' f''*. As before, constant fixation of the moving point is essential.



c. Something resembling *a* above is to be observed when the eye follows the movement of a pin head moved to and fro across the Zöllner figure in Ex. 164. As the pin goes from left to right the first, third, fifth and seventh black bands move downward, the second, fourth and sixth move upward. When the pin goes from right to left, the movement of the bands is reversed. The apparent inclination of the bands is also increased by the movement and the ordinary illusion intensified. The upward moving bands incline toward the side from which the pin starts, the downward moving incline in the opposite way. Moving the pin in a line parallel to the bands, decreases or abolishes the ordinary illusion. The illusion

of motion in the bands is evidently suggested by an illusory movement of the short oblique lines induced by the moving pin head in exactly the same way as the movement of the line *A B* in *a* above. Constancy of fixation is important here as before; some observers may find this easier to accomplish if the pin is held still and the diagram moved behind it. A certain moderate rate of movement which may easily be found by trial is best. The writer finds some help from bringing the diagram rather near the eye, *i. e.*, within six or eight inches.

The apparent movement so strongly present in this experiment is regarded by Helmholtz as the key to the explanation of the Zöllner and similar illusions. That such an illusory motion might be a factor is strongly suggested by the "unsteadiness" of the figure on casual examination. (See also *d* below).

d. Near the edge of the table drive a couple of small nails about two feet apart and lay a sheet of paper between them. On the paper and against the nails on the side away from the edge of the table lay a meter-stick or other long ruler. Take a pair of compasses of large span, say a foot; set one point close to the rule at one side and bring the other point down to the rule about the middle of the paper. Short arcs drawn with the compasses in this position would not differ much from perpendiculars to the rule. Bring the compass point to a division of the scale, fixate the division mark and then move the compass point away, and slide the rule to the right or left, at the same instant, following the division mark with the eye. The movement in each case should be three or four inches. If the rule has moved to the right and the compass point away from it, the imaginary line traced by the latter will seem a good deal curved and inclined to the left. If the rule has moved to the left, the line will appear nearly straight and inclined to the right. If the compass point approaches the rule instead of leaving it at the instant of movement, the results will be reversed, movement of the rule to the right giving inclinations to the right, and toward the left inclinations to the left. The movement must not be so fast as to prevent clear seeing of the relation of the rule and compass point. Invariable fixation of the division of the rule is important.

When this experiment is compared with *c* above, it will be observed that the inclination in this case is the same as that of the bands in Zöllner's figure when the pin is moved over them. The movement of the eye in following the pin head corresponds to that of the eye in following the rule. The heavy dark bands correspond to the imaginary line drawn by the compass point. The illusion of Zöllner's figure would, therefore, depend on the apparent motion of the bands, which in turn depends on the movement of the eyes. The dislocation of the lines in the case of the Zöllner's figure is, however, considerably less than in that of the rule and compasses, for such an excessive dislocation would make the lines appear to cross, a state of things that could not be harmonized with other parts of the perception.[1]

Helmholtz, *A*, F, 727-730 (568-571).

[1]Helmholtz thus seems to give two explanations for the same illusion. In explanation of this, he says (F, 730, (571)): "It may be found surprising that I should derive the same illusion from two causes so different in appearance. But if it is recalled that in my opinion the knowledge of the measurements of the visual field in indirect vision, rests upon experience previously had by the aid of movements, and that the present movements of regard are accompanied by similar new impressions, it is seen that the two cases are not so different as they may seem in exposition; they do not differ, except as the memory and present aspect of similar circumstances."

169. The geometrical illusions viewed with unmoved eyes. Many of the illusions considered above are much weakened, and some entirely removed when eye movements are excluded. This may be done by fixation of the eyes, somewhat better by getting the figures as after-images, and most satisfactorily of all by instantaneous illumination. Try the effect of steady fixation on the figures of Ex. 163-164.

The after-image method may be tried on the Zöllner's figure (first figure under Ex. 164), which can be made to give a strong after-image as it stands, and on any of the other figures by cutting them in narrow slits in black cardboard, and then viewing them against a bright background.

The fact that many of these illusions are still present in a certain degree when movements of the eyes are excluded, does not demonstrate that any part of them is of non-motor origin. Says Wundt (4te Aufl., II, 139): "If a phenomenon is perceived with the moved eye only, undoubtedly the influence of movement upon it is proved; but it cannot be inferred in the other way, as is now and then done, that movement is without influence on a phenomenon, that persists [with the eye] at rest." As has already been shown in Ex. 158, the experiences of the eye in motion are retained and applied to its perceptions when at rest.

Helmholtz, *A*, F, 725 ff. (567 ff.).

BIBLIOGRAPHY.

AUBERT: *A*. Grundzüge der physiologischen Optik, Leipzig, 1876.

AUERBACH: Erklärung der Brentano'schen optischen Täuschung, *Zeitschrift für Psychologie*, VII, 1894, 152-160.

BRENTANO: *A*. Ueber ein optisches Paradoxon, *Zeitschrift für Psychologie*, III, 1892, 349-358; V, 1893, 61-82.

B. Zur Lehre von den optischen Täuschungen, *Ibid.*, VI, 1893-94, 1-7.

BRUNOT: Les illusions d'optique, *Revue scientifique*, LII, 1893, 210-212.

DELBŒUF: *A*. Note sur certaines illusions d'optique, Bull. de l'Acad. roy. de Belgique, 2e série, t. XIX, 1865, No. 2, 195-216.

B. Seconde note sur de nouvelles illusions d'optique, *Ibid.*, t. XX, 1865, No. 6, 70-97.

C. Une nouvelle illusion d'optique, *Revue scientifique*, LI, 1893, 237-241.

DRESSLAR: A New Illusion for Touch and an Explanation for the Illusion of Displacement of Certain Cross Lines in Vision, AMERICAN JOURNAL OF PSYCHOLOGY, VI, 1893-95, 275.

HELMHOLTZ: *A*. Optique physiologique, Paris, 1867.

HERING: *A*. Der Raumsinn und die Bewegungen des Auges, Hermann's Handbuch der Physiologie, III, Th., i, 343-601.

B. Beiträge zur Physiologie, Leipzig, 1861-64.

JAMES: Principles of Psychology, New York, 1890.

JASTROW: *A*. A Study of Zöllner's Figure and Other Related Illusions, AMERICAN JOURNAL OF PSYCHOLOGY, IV, 1891-92, 381-398.

B. On the Judgment of Angles, *Ibid.*, V, 1892-93, 214-221.

KNOX AND WATANABE: On the Quantitative Determination of an Optical Illusion, AMERICAN JOURNAL OF PSYCHOLOGY, VI, 1893-95, 413-421, 509-514.

KUNDT: Untersuchungen über Augenmaass und optische Täuschungen, *Pogg. Ann.*, CXX, 1863, 118-158.

LÁSKA: Ueber einige optische Urtheilstäuschungen, *DuBois-Reymond's Archiv*, 1890, 326-328.

LAQUEUR: Ueber pseudentoptische Gesichtswahrnehmungen, *v. Graefe's Archiv*, XXXVI, 1890, i., 62-82. Contains historical references.

LE CONTE: *A*. Sight, New York, 1881.
B. On an Optical Illusion, *Phil. Mag.*, Ser. 4, XLI, Jan.-June, 1871, 266-269.

LIPPS: *A*. Optische Streitfragen, *Zeitschrift für Psychologie*, III, 1892, 498-504.
B. Aesthetische Faktoren der Raumanschauung. Beiträge zur Psychologie und Physiologie der Sinnesorgane (Helmholtz Festgruss), 1891, 219-307. For an abstract of this paper by Lipps himself, see *Zeit. f. Psy.*, III, 1892, 219-221.
C. Die Raumanschauung und die Augenbewegungen, *Ibid.*, III, 1892, 123-171.

MÜLLER-LYER: Optische Urtheilstäuschungen, *DuBois-Reymond's Archiv*, 1889, Supplement-Band, 263-270.

MÜNSTERBERG: Augenmass, *Beiträge zur experimentellen Psychologie*, H. 2, 1889, 125-181.

STUMPF: Ueber den psychologischen Ursprung der Raumvorstellung, Leipzig, 1873.

WALLENBERG: Der "Le Cat'sche Versuch" und die Erzeugung farbiger Schatten auf der Netzhaut, *Pflüger's Archiv*, XLVIII, 1890-91, 537-543.

WUNDT: *A*. Grundzüge der physiologischen Psychologie, Leipzig, 3te Aufl., 1887, 4te Aufl., 1893.

ZÖLLNER: *A*. Ueber eine neue Art von Pseudoskopie und ihre Beziehungen zu den von Plateau und Oppel beschriebenen Bewegungsphänomenen, *Pogg. Ann.*, CX, 1860, 500-523.
B. Ueber die Abhängigkeit der pseudoskopischen Ablenkung paralleler Linien von dem Neigungswinkel der sie durchschneidenden Querlinien, *Pogg. Ann.*, CXIV, 1861, 587-591.

APPENDIX.

THE FIELD OF REGARD AND LISTING'S LAW.

Experiment 158 of this section requires a somewhat fuller understanding of Listing's Law than can be gathered from Ex. 123, where the subject was previously treated. It has, therefore, seemed best to attempt a fuller exposition of it here.

Listing's Law as stated by Helmholtz, is as follows: "*When the line of sight passes from the primary position to any other position, the angle of torsion of the eye in its second position is the same as if the eye had come to this second position by turning about a fixed axis perpendicular both to the first and the second position of the line of sight.*"[1] On this principle rest two important corollaries: 1st. In movements from the primary position, there will be no rotation about the line of regard. 2nd. In movements from one secondary position to another, there will be some rotation about the line of regard.

The Hemispherical Field of Regard.

The usual way of putting the law to experimental test is to get a strong after-image of a rectangular cross on the centre of the eye, and then to observe the changes that its projected image undergoes as the eye is turned to one point and another of its field of regard. In the model from which the accompanying stereoscopic diagram is taken, an attempt has been made to show the changes

[1]Helmholtz, Optique physiologique, p. 606, (466); Le Conte, Sight, 174.

that such an after-image would undergo when projected upon different parts of the hemispherical field. The primary meridian of this field is $A * B$,[1] and other meridians are shown at intervals of 20°. The equator of the field (that is the line of intersection of the plane of regard with the hemispherical field of regard, when the eyes are in the primary position) is $C * D$, and above and below it are shown parallels at intervals as before of 20°. The eye itself is supposed to be at the centre of the sphere, *i. e.*, in the plane of the letters A, K, G, N, etc., and at the centre of the circle that they mark. When the eye is in its primary position, it is directed forward and fixed upon the central eight-rayed cross. Let us suppose that the eye takes a lasting after-image from the cross, but first from the horizontal and vertical rays only. If, now, the point of regard is elevated or depressed in the primary meridian, and there is no rotation about the line of regard, the vertical bar of the after-image cross will still be found to lie in the meridian; and if the point of regard be carried to the right or left in the equator of the field, the horizontal bar will still lie in the equator. This is shown by the slender crosses 40° from the centre on $A * B$ and $C * D$. The axes about which the eye turns are evidently in the plane of the letters A, K, G, N, etc., and coincide in the first case with the diameter $C\,D$, and in the second with the diameter $A\,B$. Suppose now, that the after-image has been taken from the oblique bars of the central cross, and that the movement of the eye has been oblique to the right and upward, and to the left and downward along $H * G$, and to the left and upward, and to the right and downward along $E * F$, but without rotation about the line of regard. As before, those bars of the cross which originally coincided with these lines will be found to coincide with them after the movement, as shown by the corresponding bars of the slender crosses in these positions. The axis for movements in $G * H$ lies in the diameter $E\,F$, and that for movements in $E * F$ in the diameter $G\,H$. For any intermediate directions of movement, the axes would have a corresponding intermediate position, but in all cases the axes would lie in the plane of the letters A, K, G, N, etc., perpendicular to the line of regard, both before and after its movement.

Since these after-images are always projected on a hemispherical surface there is no distortion of any of the crosses, and all of their parts maintain exactly the relations among themselves which exist among those of the central cross. It will be observed, however, that in the oblique positions the bars corresponding to the vertical of the central cross, do not quite coincide with the meridian passing through the centre of the crosses, but make a small angle with it, and that in the same way the bars corresponding to the horizontal in the central cross have no longer the same direction as the parallels above and below them. In other words the vertical and horizontal bars appear to have rotated, though the fact that the oblique bars have maintained their coincidence with the circles $E * F$, and $G * H$ shows that the rotation is not real, but as Le Conte says, "only an apparent rotation consequent upon reference to a new vertical meridian of space." This apparent rotation is known as *torsion*. The rules for this torsion are as follows: Movement of the eyes upward and to the right gives torsion to the right; upward and to the left, torsion to the left; downward and to the right, torsion to the left; downward and to the left, torsion to the

[1]In naming the curves of the hemispherical field, the asterisk (*) is used for the central cross instead of a letter.

right—all of which can easily be observed in the stereoscopic figure.

So much for movements from the primary to a secondary position. Movements from any secondary position to the primary are evidently executed about the same axes as before, but in the contrary direction. It remains then to consider movements from one secondary position to another. Let us start with an after-image from the slender cross on $C * D$, 40° to the right of the centre, and move upward along the meridian. The vertical bar of this cross coincides with the meridian at the start; when we reach the position of the eight-rayed cross, however, it no longer does so, but has turned slightly to the right—this time owing to a true rotation about the line of regard, and not to reference to a new meridian. The amount of rotation is small, in this case about 13°. Movement downward along the meridian would have exactly the same result, except that the rotation would be in the opposite direction, and similar rotations would be found if the cross 40° to left of the centre on $C * D$ had been used for vertical movements, or the crosses 40° above and below the centre on $A * B$ had been used for horizontal movement along great circles.

If movements from secondary positions along great circles are attended with this deviation of the bar of the cross from the line in which it moves, are there any lines to be found along which the eye may move the after-image without finding such a deviation? There are such lines, and four of them are shown in the figure. They are the arcs $I J$, $K L$, $M N$ and $O I$. It will be seen that these are drawn through the sloping positions of the bars corresponding to the vertical and horizontal bars of the central cross, and are perpendicular to $A * B$ and $C * D$ like the bars of the side crosses. Along these lines, a short line or after-image can be moved without leaving the line, a peculiarity in which they resemble a straight line, and when seen with the eye at rest under proper conditions they do actually appear straight. These are the Circles of Direction or Right Circles, (*Cercles de Direction, Richtkreise*) of Helmholtz.[1] The verticle circles of direction have, it will be observed, somewhat greater curvature than the meridians through the same points, and the horizontal circles of direction somewhat less than the parallels near which they lie. These circles have the further peculiarity that they all pass through the *occipital point*, a point as far behind the eye as the primary point of regard is in front of it. Both of these properties are shared also by all the great circles passing through the primary point of regard, so that they also are circles of direction. Circles of this kind, great or small as the case may be, can be passed through any two points in the field, and are not limited to those shown in the figure.

The mathematical study of Listing's Law shows that the movement from one secondary position to another may, like those from the primary position, be conceived as rotations about fixed axes all of which lie in a plane, (though in this case the plane is not perpendicular to the line of regard), and that in every case there is also a line about which there is no rotation, the *atropic line*, though this does not coincide with the line of regard.

The Plane Field of Regard.

The experimental testing of Listing's Law is generally carried out with the plane, instead of the hemispherical, field of regard,

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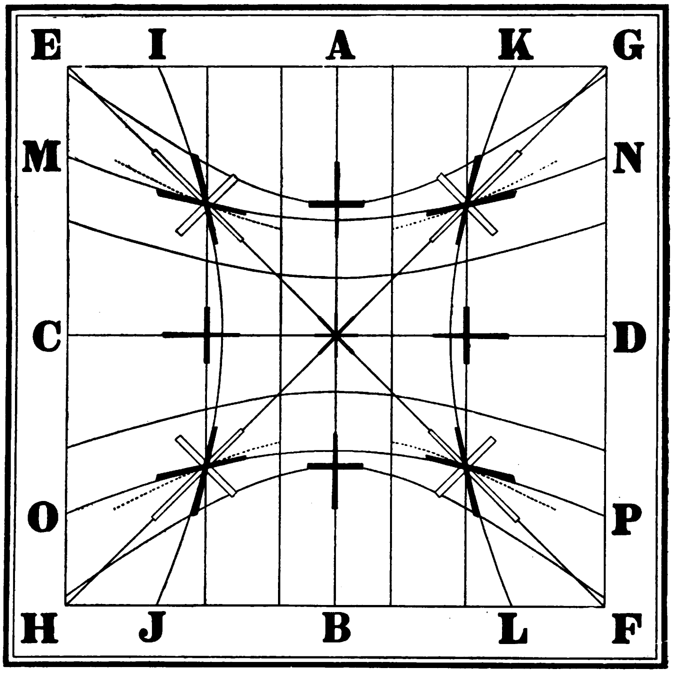
[1] *Op. cit.*, 636-637, 703-713, (492-493, 548-557.)

because of the difficulty of providing a large enough hollow hemisphere. But this has the disadvantage of adding to the changes in the after-image due to the movements of the eyes, a wholly new set of distortions due to the projection of the image upon an oblique surface. These are easily seen in the figure for the plane field.

This figure is a gnomonic projection of the hemispherical field upon a plane tangent to it at the middle point of the central cross. On this plane all the lines of the hemispherical field are represented exactly as their shadows would be cast by a point of light in the place of the eye, *i. e.*, in the centre of the sphere. The meridians are represented by vertical straight lines, wider and wider apart as they are removed from the primary meridian *A B*. The parallels become hyperbolas, increasing in curvature as they are more distant from the equator of the field. The great circles through the primary point of regard are straight lines through the same point. The other circles of direction are hyperbolas. They maintain their resemblance to straight lines, however, in so far as concerns a short linear after-image moved along them, and are called by Helmholtz the *right-lines* of the field of regard. The lettering of all the lines in the two figures is the same, so that comparison will be easy.

The distortion of the crosses on *A B* and *C D* is easy to understand, and also the oblique bars of those on *E F* and *G H*. The bars corresponding to the vertical and horizontal arms of the central cross — in solid black in the figure — require a little explanation. If the matter were one of simple projection, without torsion, the bar corresponding to the vertical ought to coincide with the projection of the meridian, and that corresponding to the horizontal bar ought to coincide with the projection of the line cutting the meridian at right angles in the hemispherical field, *i. e.*, the projection of the parallel that passes through the centre of the cross—the dotted lines in the figure. When these things are regarded it is found that both arms of the cross show torsion as in the hemispherical field, though the distortion due to projection seems at first to have turned the two arms in opposite directions.

This exposition has necessarily been physiological and geometrical. The psychological interest in the matter depends on the fact that the perception of space with the eye at rest is profoundly affected by its experiences in motion, a large group of which are received while the eye is functioning in more or less accord with Listing's Law. For a fuller account of these psychological matters, see Ex. 158.



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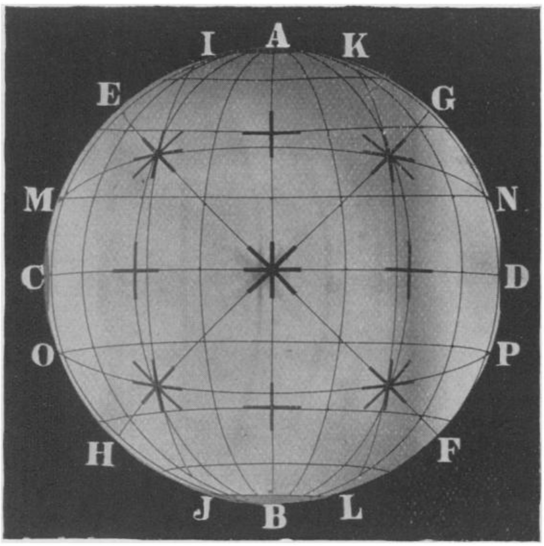

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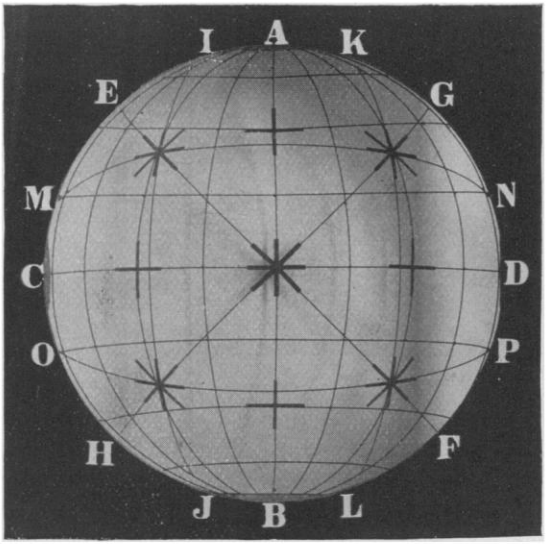

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